Exercise 3.7:

(7) Take derivative on both sides with respect to x:

$$2yy^{'} = \frac{2}{(x+1)^2}$$

Then we conclude that:

$$y^{'} = \frac{1}{y(x+1)^2}$$

(11) Take derivative on both sides with respect to x:

$$0 = 1 + \sec^2 xy(xy)' = 1 + (y + xy^{'}) \sec^2 xy$$

Then we conclude that:

$$y^{'} = -\frac{\cos^2 xy + y}{x}$$

Exercise 3.8:

(23) (a)Since $x^2 + y^2 = 25 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$, and as we know:$\frac{dx}{dt} = 3$. Then the top of the ladder sliding down the wall is:$\frac{dy}{dt} = -\frac{4}{3} \cdot 3 = -4$
(b)

$$A = \frac{1}{2}xy \Rightarrow \frac{dA}{dt} = \frac{1}{2}(x\frac{dy}{dt} + y\frac{dx}{dt}) = \frac{1}{2}(-4 \cdot 4 + 3 \cdot 3) = -\frac{7}{2}$$

(c)

$$\sin\theta = \frac{y}{5} \Rightarrow \cos\theta\frac{d\theta}{dt} = \frac{1}{5}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{5\cos\theta}\frac{dy}{dt} = -\frac{5}{5 \cdot 4} \cdot 4 = -1$$

(41) Define r to be the thickness of the ice, then the volume of the ice $V = \frac{4}{3}\pi(10+r)^3 - \frac{4}{3}\pi \cdot 10^3$ and the outer surface area of ice $S = 4\pi(r+10)^2$, then take derivative to find:

$$\frac{dV}{dt} = 4\pi(10+r)^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{160}{4\pi(10+5)^2} = -\frac{8}{45\pi}$$

$$\frac{dS}{dt} = 8\pi(r+10)\frac{dr}{dt} = -8\pi(5+10)\frac{8}{45\pi} = -\frac{64}{3}$$

1. Note that:$f^{'}(x) = \frac{1}{3}x^{-\frac{2}{3}} - \frac{1}{3}$ then $f^{'}(x) = 0 \Rightarrow x = 1$, as we see when $x > 1$ we have $f^{'}(x) < 0$, and when $0 < x < 1$ we have $f^{'}(x) > 0$. So $x = 1$ we have its global maximum on $x > 0$, or $f(x) \leq f(1) = 0$.

$$\frac{x}{y} > 0 \Rightarrow f(\frac{x}{y}) = (\frac{x}{y})^{\frac{1}{3}} - \frac{1}{3}(\frac{x}{y}) - \frac{2}{3} \leq 0$$

$$x^{\frac{1}{3}}y^{\frac{2}{3}} \leq \frac{1}{3}x + \frac{2}{3}y$$

2. (a) $f'(x) = \dfrac{x^2(x^2-12)}{(x^2-4)^2}$ and $f''(x) = \dfrac{8x(x^2+12)}{(x^2-4)^3}$ for $x \neq \pm 2$.

   (b) (i) $f'(x) > 0$ when $x < -2\sqrt{3}$ or $x > 2\sqrt{3}$.

   (ii) $f'(x) < 0$ when $-2\sqrt{3} < x < 2\sqrt{3}$.

   (iii) $f''(x) > 0$ when $-2 < x < 0$ or $x > 2$.

   (iv) $f''(x) < 0$ when $x < -2$ or $0 < x < 2$./

   (c) By (b), $(-2\sqrt{3}, -3\sqrt{3})$ is a local maximum, $(2\sqrt{3}, 3\sqrt{3})$ is a local minimum and $(0,0)$ is a saddle point.

   (d) By (b) again, $(0,0)$ is a point of inflextion.

   (e) Note that $\lim_{x\to 2^-} f(x) = -\infty$ and $\lim_{x\to 2^+} f(x) = +\infty$, also $\lim_{x\to -2^-} f(x) = -\infty$ and $\lim_{x\to -2^+} f(x) = +\infty$. Therefore, $x = 2$ and $x = -2$ are vertical asymptotes.

   Note that $m = \lim_{x\to\infty} \dfrac{f(x)}{x} = 1$ and $\lim_{x\to\infty} f(x) - mx = 0$. Therefore, $y = x$ is the oblique asymptote.

   (f) The graph of $f(x)$.